

NAG Toolbox for MATLAB

f07qs

1 Purpose

f07qs solves a complex symmetric system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by f07qr, using packed storage.

2 Syntax

```
[b, info] = f07qs(uplo, ap, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07qs is used to solve a complex symmetric system of linear equations $AX = B$, the function must be preceded by a call to f07qr which computes the Bunch–Kaufman factorization of A , using packed storage.

If **uplo** = 'U', $A = PUDU^T P^T$, where P is a permutation matrix, U is an upper triangular matrix and D is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 blocks; the solution X is computed by solving $PUDY = B$ and then $U^T P^T X = Y$.

If **uplo** = 'L', $A = PLDL^T P^T$, where L is a lower triangular matrix; the solution X is computed by solving $PLDY = B$ and then $L^T P^T X = Y$.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates how A has been factorized.

uplo = 'U'

$A = PUDU^T P^T$, where U is upper triangular.

uplo = 'L'

$A = PLDL^T P^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **ap**(*) – complex array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The factorization of A stored in packed form, as returned by f07qr.

3: **ipiv**(*) – int32 array

Note: the dimension of the array **ipiv** must be at least $\max(1, n)$.

Details of the interchanges and the block structure of D , as returned by f07qr.

4: **b(ldb,*)** – complex array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs_p})$

The n by r right-hand side matrix B .

5.2 Optional Input Parameters1: **n** – int32 scalar

Default: The first dimension of the array **ap** and the second dimension of the array **ap**. (An error is raised if these dimensions are not equal.)

n , the order of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – int32 scalar

Default: The second dimension of the array **b**.

r , the number of right-hand sides.

Constraint: $\mathbf{nrhs_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldb

5.4 Output Parameters1: **b(ldb,*)** – complex array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs_p})$

The n by r solution matrix X .

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **nrhs_p**, 4: **ap**, 5: **ipiv**, 6: **b**, 7: **ldb**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if **uplo** = 'U', $|E| \leq c(n)\epsilon P|U||D||U^T|P^T$;

if **uplo** = 'L', $|E| \leq c(n)\epsilon P|L||D||L^T|P^T$,

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_{\infty}}{\|x\|_{\infty}} \leq c(n) \operatorname{cond}(A, x) \epsilon$$

where $\operatorname{cond}(A, x) = \| |A^{-1}| |A| |x| \|_{\infty} / \|x\|_{\infty} \leq \operatorname{cond}(A) = \| |A^{-1}| |A| \|_{\infty} \leq \kappa_{\infty}(A)$.

Note that $\operatorname{cond}(A, x)$ can be much smaller than $\operatorname{cond}(A)$.

Forward and backward error bounds can be computed by calling f07qv, and an estimate for $\kappa_{\infty}(A)$ ($= \kappa_1(A)$) can be obtained by calling f07qu.

8 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This function may be followed by a call to f07qv to refine the solution and return an error estimate.

The real analogue of this function is f07pe.

9 Example

```

uplo = 'L';
ap = [complex(-0.39, -0.71);
      complex(-7.86, -2.96);
      complex(0.5278724801640799, -0.3714660014825906);
      complex(0.442558238872675, +0.1936483698297402);
      complex(-2.83, -0.03);
      complex(-0.6078391056683192, +0.281079647893122);
      complex(-0.4822822975185383, +0.01498936219105284);
      complex(4.407906236731014, +5.399120676796941);
      complex(-0.1070821880092683, -0.3156780862488454);
      complex(-2.095414887840057, -2.201139281440786)];
ipiv = [int32(-3);
        int32(-3);
        int32(3);
        int32(4)];
b = [complex(-55.64, +41.22), complex(-19.09, -35.97);
     complex(-48.18, +66), complex(-12.08, -27.02);
     complex(-0.49, -1.47), complex(6.95, +20.49);
     complex(-6.43, +19.24), complex(-4.59, -35.53)];
[bOut, info] = f07qs(uplo, ap, ipiv, b)

bOut =
    1.0000 - 1.0000i   -2.0000 - 1.0000i
   -2.0000 + 5.0000i    1.0000 - 3.0000i
    3.0000 - 2.0000i    3.0000 + 2.0000i
   -4.0000 + 3.0000i   -1.0000 + 1.0000i
info =
      0

```